Partially simple graphs form a quasitopos

Aloïs Rosset

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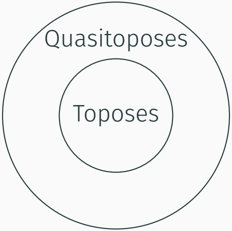

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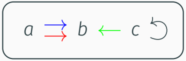
We will look at two categories: Set and (directed) Graph



Subobject classifier

Definition

Subobject classifier is $\mathrm{True} : 1 \to \Omega$ with

$$\begin{array}{ccc} A \subseteq B & \iff & \chi_A : B \to \Omega \\ \text{subobjects} & & \text{characteristic function} \end{array}$$

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- Set : $\Omega \coloneqq \{0, 1\}$

$$A \subseteq B \iff \chi_A : b \mapsto \begin{cases} 1, \text{ if } b \in A \\ 0, \text{ otherwise} \end{cases}$$

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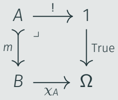

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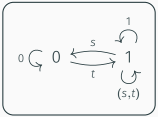
- Graph : $\Omega \coloneqq$ 

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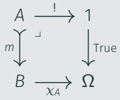

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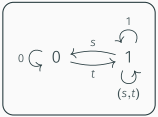
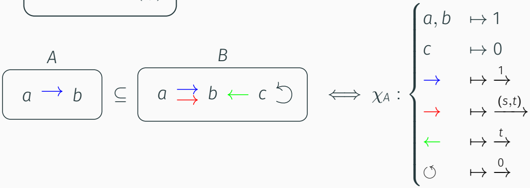
- Graph : $\Omega \coloneqq$ 



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Logical connectives:

$$\begin{aligned} \text{False} &\coloneqq \chi_{0 \overset{!}{\rightarrowtail} 1} \\ \neg &\coloneqq \chi_{\text{False}} \\ \wedge &\coloneqq \chi_{\langle \text{True}, \text{True} \rangle} \\ \Rightarrow &\coloneqq \chi_{\text{Eq}(\pi_1, \wedge) \to \Omega \times \Omega} \\ \vee &\coloneqq \chi_{[\langle \text{True}_\Omega, \mathrm{id}_\Omega \rangle, \langle \mathrm{id}_\Omega, \text{True}_\Omega \rangle]} \end{aligned}$$

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(for object X, then $\text{True}_X \coloneqq X \xrightarrow{!} 1 \xrightarrow{\text{True}} \Omega$ means true *in context* X)

LT-Topologies

Lawvere-Tierney Topologies

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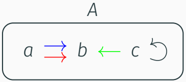
Induces ***closure*** operator on subobjects: $\overline{A_0 \rightarrowtail A}$: $\quad \chi_{\overline{A_0 \rightarrowtail A}} := j \circ \chi_{A_0 \rightarrowtail A}$.

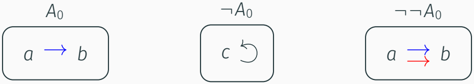
Subobject $A_0 \rightarrowtail A$ is **dense** if $\overline{A_0} = A$.

Examples of topologies

	Topology j	Closure $\overline{A_0 \subseteq A}$	Dense object
Set, Graph	*Trivial* True_Ω	A (adds everything)	all
Set, Graph	*Discrete* id_Ω	A_0 (adds nothing)	only A
Graph	*Closed for st* $(- \vee st)$	$A_0 \cup A(V)$ (adds all vertices)	if $A_0(E) = A(E)$
Graph	*Dbl. negation* $\neg\neg$	$A_0 \cup \big(A(E) \cap (A_0(V) \times A_0(V))\big).$ (adds all valid edges)	if $A_0(V) = A(V)$

Double negation topology

¬¬-topology on Graph: for 



Sep. & Sheaves

Separated elements

Standard topology: separated (Hausdorff) space B has property that any $f : A \to B$ is fully determined by its image on any dense subsets of A.

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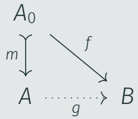
Definition

Topology j, object B, arbitrary dense $m : A_0 \rightarrowtail A$ and arbitrary $f : A_0 \to B$.
Count the number of factorisations $g : A \to B$ of f through m $(\forall A_0, A, f)$:

- B is ***separated*** if $\#g \leqslant 1$,
- B is ***complete*** if $\#g \geqslant 1$,
- B is a ***sheaf*** if $\#g = 1$.



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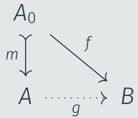

Lemma (Johnstone '79)

For a topology, separated elements and sheaves form two quasitoposes.

Simple graphs via topologies (Both directed and undirected)

Lemma (Vigna '03)

Simple graphs are the $\neg\neg$-separated elements and form thus a quasitopos.

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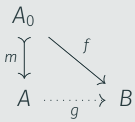
Proof.

Subgraph $A_0 \subseteq A$ is ¬¬-dense if A_0 has all vertices.

Then $\#g = \#$ way to maps edges of A onto edges of B.

This is $\leqslant 1$ iff B has at most one edge between each pair of vertices.



□

Separated elements and sheaves in Set and Graph

Top.	Closure $\overline{A_0 \subseteq A}$	Dense object	Sep. elem.	Sheaves
Triv.	(adds everything)	all	subterminal objects	terminal objects
Dis.	(adds nothing)	only A	(all)	all
Closed	(adds all vertices)	if $A_0(E) = A(E)$	$\varnothing$,  $\cdots$	 $\cdots$
$\neg\neg$	(adds all valid edges)	if $A_0(V) = A(V)$	simple graphs	complete graphs

BiColGraph

$$\mathrm{Graph} = \mathrm{Set}^{I^{\mathrm{op}}} \text{ for } I^{\mathrm{op}} = \quad E \underset{t}{\overset{s}{\rightrightarrows}} V \; .$$

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I.e., graphs with edges partitioned into two sets: blue and red.

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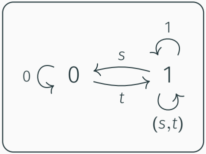
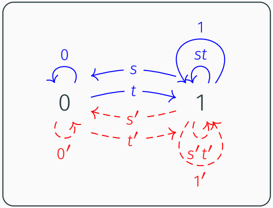
Ω_{Graph}



$\Omega_{\mathrm{BiColGraph}}$



Topologies on BiColGraph

Lemma

4 *topologies on* Graph $\implies$ 8 *topologies on* BiColGraph.

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	j_1	j_2	j_3	j_4	j_5	j_6	j_7	j_8
on E	*disc.*	$\neg\neg$	*disc.*	$\neg\neg$	*closed*	*closed*	*triv.*	*triv.*
on E′	*disc.*	*disc.*	$\neg\neg$	$\neg\neg$	*closed*	*triv.*	*closed*	*triv.*

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j_2-sep. el. = blue-simple graphs.
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$\implies$ Partially simple graphs.

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$$\left.\begin{array}{l} j_2\text{-sep. el.} = \text{blue-simple graphs.} \\ j_3\text{-sep. el.} = \text{red-simple graphs.} \end{array}\right\} \implies \text{Partially simple graphs.}$$

Corollary

Partially simple graphs form a quasitopos.

Fuzziness

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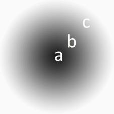

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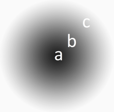

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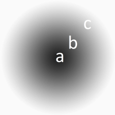

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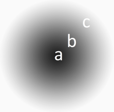

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More general definition: poset $(\mathcal{L}, \leqslant)$ instead of $[0, 1]$.

Why fuzzy structure?

See membership values as **labels** $v_1^x \xrightarrow{e^y} v_2^z$.

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$(\mathcal{L}, \leqslant)$ complete lattice $\Longrightarrow$ practical **graph relabelling**

(join $\bigvee$, meet $\bigwedge$, bot $\bot$, top $\top$) (with PBPO$^+$ formalism)

$$v_1^x \xrightarrow{e} v_2 \quad \xRightarrow[\text{(impossible with DPO)}]{\text{single PBPO}^+ \text{ rule}} \quad v_1^y \xrightarrow{e} v_2$$

(Quasi)topos	Graph	BiColGraph	Fuzzy graphs
Quasitopos (¬¬-separated elements)	Simple graphs	Partially simple graphs	Simple fuzzy graphs

All topologies on FuzzySet($\mathcal{L}$)

Lemma

Topologies on FuzzySet($\mathcal{L}$) $\iff$ $1 + \{\phi : \mathcal{L} \to \mathcal{L}$, *increasing, monotone, idempotent,* $\phi(x) \wedge y \leqslant \phi(x \wedge y)\}$

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- Funnily, $\neg\neg : \mathcal{L} \to \mathcal{L}$ gives *another* topology

  (if $\mathcal{L}$ is a De Morgan Heyting Algebra)

All topologies on $\mathsf{FuzzySet}(\mathcal{L})$

Lemma

Topologies on $\mathsf{FuzzySet}(\mathcal{L})$ $\iff$ $1 + \{\phi : \mathcal{L} \to \mathcal{L}$, *increasing, monotone, idempotent,* $\phi(x) \wedge y \leqslant \phi(x \wedge y)\}$

Remark. "increasing, monotone, idempotent" $\iff$ ϕ is a monad.

Examples

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Lemma

Similarly, topologies on $\mathsf{FuzzyGraph}(\mathcal{L})$ *are a combinations of topologies on* Graph *and of functions* $\phi_V, \phi_E : \mathcal{L} \to \mathcal{L}$.

Separated elements in FuzzySet($\mathcal{L}$)

Lemma

For a topology on FuzzySet($\mathcal{L}$) *corresponding to some* $\phi : \mathcal{L} \to \mathcal{L}$*:*

- *every fuzzy set is separated,*
- *fuzzy set* (B, β) *is a sheaf iff* $\mathrm{im}(\beta) \subseteq \mathrm{im}(\phi)$*.*

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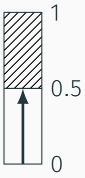
Example: $\mathcal{L} \coloneqq [0, 1]$, $\phi(x) \coloneqq \max\{x, 0.5\} = x \vee 0.5$
A fuzzy set is a sheaf if its membership are in $[0.5, 1]$.



Future work

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Negation $\neg a$ is largest element with $a \wedge \neg a = \bot$.

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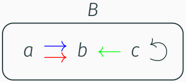

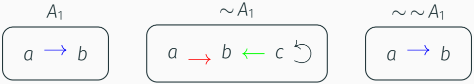

Conegation seem to be more of an *interior* operation.

Questions: Does it capture local falsehood? Does it satisfy the dual axioms? Can we obtain other new quasitoposes?

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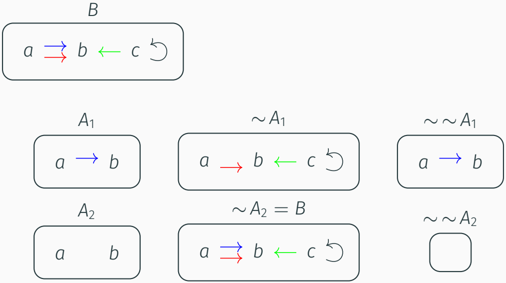

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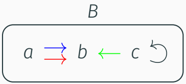

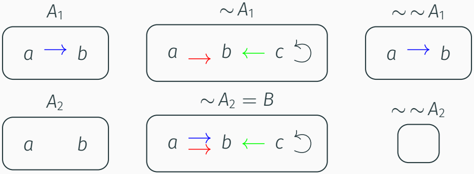

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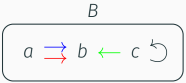

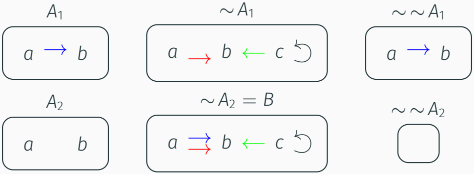

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Future questions

Other tasks:

- Obtain more quasitoposes via topologies.
- Specifically, looking at other presheaf categories (they are toposes).
- Can we deduce the internal logic of the quasitopos obtained from the internal logic of the topos?